THE SURVEYOR'S COMPANION.

A

Manual for Practical Surveyors,

CONTAINING

METHODS INDISPENSABLY NECESSARY

FOR

ACTUAL FIELD OPERATIONS,

BY E. W. BEANS,

Norristown, Montgomery Co., Pa.

PHILADELPHIA:

PUBLISHED BY J. W. MOORE,

No. 195 Chestnut Street.

1854.

PRINTED BY ISAAC ASHMEAD.

PREFACE.

The want of a work on Practical Surveying has been long felt, and is generally acknowledged. The numerous publications on surveying would seem to preclude the necessity of anything new. But upon examination we find the wants of the student have been consulted, rather than those of the practical man. Indeed, many of the publications in general use appear to have been written by those who were engaged in the instruction of youth, and who were unacquainted with the practical part of surveying, excepting perhaps so far as may have been requisite for the information of the classes under their direction.

I have conversed with many persons who

have been extensively engaged in land surveying, and I have not in a single instance met with any one who has not expressed his unqualified conviction of the want of a work adapted to the purposes of the practical surveyor. Within a few years there have been published several works on surveying: amongst them may be mentioned that by John Gummere, a treatise which cannot be too strongly recommended to those who wish to become familiar with this subject; and had it been as well adapted to the *wants* of the *practitioner*, as to those of the student, no other need have been desired. Professor Davies' "Elements of Surveying" is an excellent performance. Flint's "Surveying" also contains much useful practical information. But still there seems to be wanted a more minute detail of expedients employed in the field.

With a view to supply this defect, the

following pages have been written, designed as a suitable treatise to be placed in the hands of those who wish to become familiar with the *practice* of surveying. A systematic arrangement has not been followed; but as my object is to supply the wants of the practical man, (those of the student having already been supplied by the authors mentioned,) this will be a matter of minor importance.

In submitting the work to the public, it is not pretended to be complete in itself, but only introductory to a subject of much importance hitherto almost entirely neglected. If it prove a useful auxiliary to those who are about assuming the responsible duties of practical surveyors, the object of the publication will have been accomplished.

E. W. BEANS.

Norristown, Montgomery county,
Pa., June 1853.

A MANUAL FOR SURVEYORS.

CHOICE OF INSTRUMENTS.

It is of the first importance to every young person, about commencing the practice of Surveying, to furnish himself with suitable instruments. These should be of the very best character, both as regards the workmanship and their adaptation to purposes in which they are intended to be employed. His success may depend upon this choice, his pretensions to accuracy, in what he may undertake, must rest very much upon it. Unless he is acquainted with the use of the Instruments in the field, it would be well to advise with some person in whose judgment he can confide. It would also be advisable to visit the shops of the different instrument makers, and examine the various instruments in use. He may be enabled by these means

to make a more suitable choice than he would otherwise do. Having the instruments before him, and their peculiar advantages and uses explained, he would be enabled to select those best suited to his purpose. He should be provided with an instrument for measuring angles, and taking bearings, a chain, measuring-tape, plummet, reading-glass, &c. If he intends laying off town lots, he should have a leveling instrument and rod, also a 20 feet frame for measuring distances accurately.

The two pole chain is commonly used. It should be made of heavy wire, the links connected by double rings to prevent the chain kinking, it should be firm in all its parts, and of the standard length.

The compass or circumferenter is in general use amongst surveyors in this country. In selecting a compass, particular attention should be paid to the needle. If it vibrates long and settles with a gradual diminution in the arc of vibration, it moves with freedom on the centre pin; but if it settles after a few vibrations or suddenly at any point, it does not move with sufficient freedom on

its centre, or is defective, and should not be used on valuable lands.

To examine the divisions of the limb of the instrument, set the needle to any degree; if both ends coincide with the corresponding divisions on the opposite limbs of the compass, it shows the adjustment is correct on that course. In the same manner examine various divisions round the compass box, the coincidence of the opposite ends of the needle, with the same degree, is a proof of its being correct. If it be a nonius compass, the nonius may be moved to the right or left a few degrees; if the needle move over the same number of degrees, it is a proof of the correctness of the graduations. The instrument may be further tested by laying off, with it, on the ground, a square, the side of which may be 50 or more perches. Then if the diagonals be measured and found equal (or very nearly so, as errors may be in the measured distances,) it is evidence of the correctness of the divisions, needle, &c., and that the instrument is a good one. Any regular figure may be used for this purpose. A nonius compass

is preferable to a plain one in several particulars—some of which may be mentioned. Having found the difference of variation between the present time, and that at which a survey was formerly made, set the nonius to this difference, the needle will show, on the face of the instrument, when the sights are set to the lines of survey, the same course as formerly, unless the compasses, by which the bearings were taken, differ in other particulars. Another reason for preferring the nonius compass is, that we may run a line to minutes, by setting the needle to the whole degrees, and the nonius to the minutes of the course.

Angles of elevation, when a level is attached to the compass, (which it should always have,) may be taken, by having one of the sights graduated into degrees. This may be easily done by the surveyor himself calculating the tangents of the angles, to a radius, equal to the distance of the sights apart, and setting off those distances, on the sight. This may be carried to about 20°. The links to be deducted, for every chain length oblique measure, to reduce it to horizontal measure, may

also be marked on the sights, corresponding with the degrees of elevation. A compass so fitted up is very useful in hilly grounds; for having an object at the top of the hill, we may easily take the angle of elevation of the hill, and thence reduce the oblique line measured, to the horizontal line.

Horizontal angles may be measured with the nonius compass. For this purpose, procure an extra pair of sights, made so as to be attached to the face or movable part of the instrument containing the needle, and at right angles with the fixed sights. Bring the fixed sights to bear on an object in one of the lines, including the angle to be measured, and clamp them in that position. Unclamp the nonius plate, to which the attached sights are affixed, and bring these sights to bear on an object in the other line, including the required angle. The number of degrees over which the vernier plate has moved, gives the difference between the angle measured and a right angle, and must be added to or subtracted from a right angle, according as the movable sights move from, or towards the fixed ones.

An arm may be also temporarily attached to the ball or spindle of the staff head, extending to the edge of the compass box, which should be divided up to 90°. This arm being clamped at 0° by means of a thumb screw, connecting the arm to the spindle or head of the staff, will point out the number of degrees the compass box may move over, as the motion of the compass is about the immovable spindle of the staff head or axis of the instrument. A nonius compass fitted up as here directed, will be nearly as efficient as a theodolite, in the ordinary cases of farm surveying; and especially where the needle may be rendered almost useless from local attraction, or other accidental occurrences.

A nonius compass may be used where there is local attraction, as follows: Bring the sights to bear on an object at the back station, then move the nonius plate until the needle settles at the same course as at the last station, and clamp the nonius plate; a course may then be taken in any direction with the needle, as correctly as if free from the effects of local attraction.

The following description of an instrument for surveying was received from Enoch Lewis, a practical surveyor, and author of several mathematical works.

"A circumferenter for surveying in the usual way, and for taking horizontal angles, is formed with two circular plates, the lower of which is firmly attached to the stem of the instrument, and may therefore be clamped to the tripod which supports it; the upper plate is connected with the sights, and moves with them. These circles are concentric, and are firmly attached by a screw, or moved the one on the other by a rack wheel. The circumference of the lower circle is graduated, and the upper one contains a nonius.

"To take an angle, place the middle of the nonius in conjunction with the zero of the lower plate, and fix them together by the screw; then direct the sights along one of the lines which include the angle in question, and clamp the lower plate to the tripod. Loose the plates from each other, and applying the eye to the sights, turn them by means of the rack wheel till they coincide with the other

line, including the angle. The angle may then be read on the fixed plate by means of the nonius.

"An instrument constructed in this manner is very convenient in laying out the curves of railroads.

"A small telescopic tube fixed on the side of the instrument parallel to the line of sights, and moveable on the centre of a graduated vertical circle, will serve to take angles of elevation or depression with great facility.

"A small table of versed sines to a radius one, or unity, extending to fifteen or twenty degrees, inserted on a leaf of the note-book, may answer the purpose of reduction from oblique to horizontal measure."

It is generally recommended, when the compass is not in use, to place it in its box, where it will not be disturbed, and let the needle settle, and remain on its centre pin. However, I believe it preferable to let the needle settle, and then carefully screw it up or off the centre pin; it will then be very nearly in the magnetic meridian, and will have all the advantages of its remaining on the centre pin,

without the danger of blunting it by continual friction on the point.

The cross is a very useful instrument to lay off perpendiculars when running lines or measuring irregular pieces of ground. A block of hard-grained wood, three or four inches square, and one and a half or two inches thick, having two saw kerfs cut more than half through its thickness, and intersecting each other at right angles at the centre of the block, will be sufficiently exact for the purposes above-mentioned; which will also be of convenient size to carry in the pocket.

To use this cross, we have only to lay it on the face of the compass adjusted for observation, and directing one of the kerfs to an object; a stake set in the direction of the other kerf will be at right angles with a line joining the cross, and the object observed.

The Graphometer, or Semicircle, is also a very useful instrument in surveying. It consists of a graduated semicircle, with a pair of sights in the direction of the diameter of the instrument, and having a pair of movable sights moving on the

centre of the semicircle, to which the verniers are attached. Underneath the centre of the instrument is a ball and socket to attach it to the stand when in use.

To measure an angle, turn the fixed sights until you see an object in the direction of one of the lines between which the angle is to be measured, then turn the movable sights until an object is seen in the direction of the other line, the vernier will point out the angle between them. The socket has a notch in one side, so as to enable the suveyor to take vertical angles or altitudes. It is a very useful instrument for many purposes in surveying.

In surveying valuable lands, or when great accuracy is desired, a theodolite or transit should be used. A description of the former is deemed unnecessary, as this has been given in some of our best treatises on surveying, and also in "Simms' Treatise on the Principal Instruments employed in Surveying, Leveling, and Astronomy," a work which every practical mathematician should possess.

The "Transit" may not be so generally known.

It consists of two parallel plates attached to each other in a manner somewhat similar to the circular plates of a theodolite. To the upper of these circular plates is attached a compass-box of much larger dimensions than that usually attached to the theodolite, which enables the observer to read the bearings from the needle much more correctly.

The circular plates are not graduated at their outer edges, but so as to be read within and at the bottom of the compass-box. To the upper plate is attached the wyes, or vees, which support the axis of the telescope, that revolves in the same manner as the astronomical transit. A vertical arc is sometimes added to measure vertical angles. The instrument is furnished with clamps, tangents, screws, and tripod, similar to the theodolite. It may be used as a circumferenter in taking bearings, or as a theodolite for measuring angles. The adjustments are few, simple, and readily made. It is usually provided with only one vernier; two, however, would be conducive to accuracy, as a mean of the readings would correct the eccentricity of the instrument. The transit may be considered,

for the general purposes of surveying, superior to any other instrument in use.

There is one property of the magnetic needle, (when not disturbed by local attraction, &c.,) which should not be lost sight of, and that is, that any error committed in running one line is not communicated to another. But when angles are taken, the errors may affect or be communicated to others, even at a remote distance from the line on which the error is committed. Therefore the accuracy of the angles must be carefully ascertained by comparison with the courses shown by the needle, otherwise great and perplexing errors may be introduced.

The surveyor should be furnished with tables of sines, as far as 20° extending to four decimals at least; nat. tangents as far as 10°, and such others as may be needful for reference in the field. These, with the Traverse Table, will be requisite to facilitate many calculations necessary to be made in the field. Scribner's Engineer's Pocket Table Book may be particularly recommended. He should

also have a reading-glass of considerable power, to aid him in reading off the bearings, angles, &c., with precision.

LAND MARKS.

Judge Wilson gave his decision in regard to lines and land marks, as follows:

The best evidence is,

1st. Living marks, such as trees, &c., the first and most substantial land marks; and if marked trees should not be in a right line, yet the line must be run from one marked tree to the next, and thence to the next, and so on.

2d. When there are stones of long standing along the line in question, the line must be run from the first to the second; from the 2d to the 3d, &c.

3d. Old residents in the neighborhood, may designate marks or points where the original line formerly run.

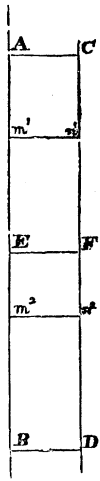
Lastly, the chain and compass.

It is the practice where there are ditches along the line, to take for the line the edge of the ditch lying next the bank of dirt thrown out in digging it, or if the dirt thrown out in digging be on both sides, the middle of the ditch must be taken.

PRACTICAL SURVEYING.

PROPOSITION 1.

To run a line between two points A and B; or to trace a right line A B on the ground.



Case 1. When the points A and B can be seen from each other:

Place the instrument over A, and bring the sights to bear on B; then direct marks to be set at the required points m^1, m^2, &c., along the line, and it is done.

N. B. In this and the following propositions, I have used n^1, n^2, n^3, m^1, m^2, &c., to denote the number of the stakes set in the lines, reckoning from the beginning.

Case 2. When obstacles prevent the points A and B being seen from each other:

On the same side of A B, and perpendicular to it set off equal distances A C and B D; place the

instrument over one of these points C, and bring the sights to bear on the other point D: then direct stakes to be set, along the line C D, opposite the points required at n^1, n^2, &c.; and from these set off perpendicular to C D, and equal to A C or B D, the distances $n^1\ m^1$, $n^2\ m^2$, &c., towards A B, then A $m^1\ m^2$ B will be the line required.

Case 3. When the points A and B can be seen from an intermediate point E:

By trials set the instrument at a point E in a right line between A and B; then the intermediate points between A E and E B may be found as in case 1.

Case 4. When obstacles prevent A and B being seen from an intermediate point, but C and D, (case 2d,) may be seen from the intermediate point F:

Set the instrument at F, the intermediate point in a right line with C and D, and direct stakes to be driven in the line C D as in the preceding case; distances perpendicular to C D, and equal to A C or B D being set off towards A B will determine the points m^1, m^2, &c., in the line A B.

PROPOSITION 2.

To run a right line on the ground, or to prolong the line A B to any distance required:

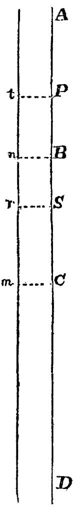
Place the instrument over A, the point from which the line is to be run. Having adjusted the instrument, bring the sights to bear on B, a staff placed in the direction of the line to be run. Let the needle down on the centre pin, and take the bearing of the line, screw up the needle carefully; next remove the instrument to B, set the needle to the course of A B and run to C, and from thence to D, and so on as far as required. When it is required to run the line accurately, proceed thus: Plant the instrument at B, and having adjusted it, bring the sights to bear on the back station A; and direct a stake forward to be set at C; remove to C, and take a back sight to B, then direct a stake to be set at a forward station D; and so proceed as far as required.

This method of running a line by back stakes, that is by having a stake set up at the last station, and bringing the sights to bear on it, should always

be adopted, unless the lines to be run are very short. The needle should not be depended upon, except where it cannot be dispensed with, as in taking courses, bearings of objects, &c.

I have used the term "sights," in order that the methods pointed out may be applicable to the use either of the circumferenter, theodolite or transit.

Case 2. When there are obstacles which prevent the line being run directly from A to D:



Run the line from A to B as before directed, where we will suppose an obstacle is met with, which prevents the line being continued.

Set off at B, perpendicular to A B any distance, B n sufficient to pass the obstacle at B; from a point P between A and B set off another perpendicular $P\,t$=B n; prolong $t\,n$ towards m, until the obstacle at B is passed. From r and m in the line $t\,n$ produced, set off the perpendiculars thereto, $r\,s$=m C=B n from r to s and m to C prolong s C to D, which will be a point in the line A B produced.

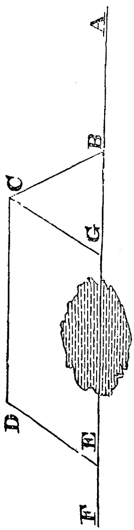
Case 3. When it is impracticable on account of intervening obstacles, to lay off perpendiculars $t\ P$ n B, &c., from the line A D:

Run $P\ t$ any suitable course; then run B n parallel to it, and make B $n = P\ t$. Prolong $t\ n$ to r and m as in the preceding case; and run $r\ s$, m C parallel to $P\ t$ and each of them equal to it; produce s C to D as in the last case, so will D be a point in the line A B produced.

Note.—A line run from a given point which does not terminate at the point designed, but falls either to the right or left of it, is called a random line, or a guide line.

Case 4. Where swampy ground, &c., prevents measuring, as well as running in the direction A B, further than to B:

On arriving at B, deflect the line B C with an angle of 60°, that is, make the angle A B C equal to 120°, and measure B C. At C deflect C D with an angle of 60°; that is, make the angle B C D = 120°, and measure C D. At D deflect the line D E with an angle of 60°, or make the angle C D E = 120°, and make D E = B C. At E deflect E F



with an angle of 60°, or make the angle D E F = 120°. The points E and F will be in the line A B produced.

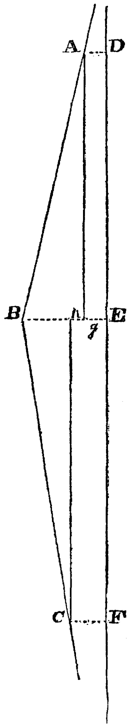
Make B G = B C and join C G, the triangle B C G is equilateral; also C G and D E are parallel, there fore D E G C is a rhomboides, and E G is equal to C D. Consequently, A E is equal to the sum of the distances A B, B G, G E, which are equal to A B, B C and C D added together.

PROPOSITION 3.

To measure an angle A B C accurately, where B C deflects but a few degrees from the line A B:

From a point D opposite to A, a point in one of the lines, including the angle to be measured, run the line D E F till we arrive at the point F, opposite C a point in the other line B C including the required angle. At E opposite the angular point B, measure the perpendicular B E, also mea-

sure the perpendiculars A D and F C; and also measure D E and E F.



Draw A g and C h parallel to D F.

Then B g=B E—E g=B E—A D

B h=B E—E h=B E—C F

Nat. Tang. angle B A g=B g ÷ Ag = B g ÷ D E. Also, nat. tang. angle B C h is equal to B h ÷ C h = B h ÷ E F.*

The sum of the angles B A g and B C h subtracted from 180°, gives $\angle$ A B C.

This method of finding the angle A B C is applicable to measuring little deviations from right lines in streets, &c., where the utmost precision is required that the buildings may be regulated exactly to the line of the street. A transit or theodolite, should be used in this case to run out the line D E F, and the distances A D, B

* If C F = A D we may say E F : F D : : $\angle g$ A B : $\angle h$ C B the angles being very nearly reciprocally proportional.

E, C F, &c., should be measured to the hundredth part of a foot.

It has not been thought proper to introduce methods of measuring angles with the instruments which have been described, as that has been pretty fully done in the works to which I have already referred. My object in the present performance is to supply what has been omitted. In doing this, I have introduced but little to be found in the authors alluded to. It may not be improper to introduce here the method of verifying the correctness of an angle by the principle of repetition. Place the transit over the angular point B (see the preceding figure,) after having adjusted it for observation, set the vernier to zero, and bring the telescope to bear on the staff at A in one of the lines, including the angle to be measured; clamp the lower plate to the tripod, unclamp the upper plate, and bring the telescope to bear on C a staff in the other line comprehending the required angle; the vernier will point out the measure of the angle A B C. In this position clamp the plates together, unclamp the lower plate from the tripod, and bring

the telescope to bear on A, by revolving the instrument bodily on its axis. Now clamp the lower plate; unclamp the upper plate, and bring the telescope to bear on C; the vernier will show twice the angle A B C. We may repeat this operation at pleasure, the last reading of the vernier being divided by the number of times the angle had been measured, will give a mean result more to be relied on than any single observation, however carefully made.

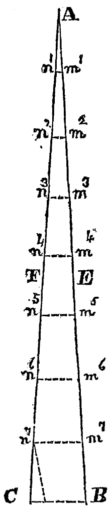
It is best to repeat the measure of an angle, that the vernier may pass over the entire circumference of the graduated plate, in which case 360° must be added to the last reading of the vernier, previous to dividing by the number of observations.

PROPOSITION 4.

Random Lines.

To run a right line between two given points, when several intermediate stations have to be taken on account of intervening obstacles.

Case 1. When the line can be run from the place of beginning:



Having adjusted the instrument at A, the point from which the line is to be run, and directed the sights towards B, as nearly as can be guessed at, this being the point to which the line is to be run from A, direct stakes to be set at every 20 perches (or otherwise, as may suit the nature of ground,) in the direction of the sights, which let be designated in the order in which they are placed by n^1, n^2, n^3, n^4, which being continued until we arrive at C, so that C B may be perpendicular to A C. The line A C in the direction of the sights, must be carefully run by Proposition 2d, Case 1st, Measure C B. Let $n^1\,m^1$, $m^2\,n^2$, $n^3\,m^3$, &c., be drawn parallel to C B. The triangles A B C, A $m^1\,n^1$, A $m^2\,n^2$, &c., being similar we have,

$$AC : An^1 :: BC : n^1 m^1$$

$$AC : An^2 :: CB : n^2 m^2, \text{ \&c.}$$

$$AC : AF :: CB : FE.$$

That is, as the whole distance measured from A to C, is to any part of the line measured A n^1, or A F; so is C B the distance from the termi-

mination of the random line, or line run from the point at which it should have terminated, to the distance from (the line measured,) the points, m^1 m^2, E, &c., in the true line A B.

Having found $n^1 m^1$, we have $n^2 m^2$, equal to twice $n^1 m^1$; also, $n^3 m^3$, equal to three times $n^1 m^1$, &c; for as the stakes are equidistant from each other, the several offsets are some multiple of the first.

1. As an example, suppose it is required to run a line between two farms having a stone at which to commence, and also another to which to run. Having run the random line as directed, and leaving stakes at every 20 perches distance from the beginning of the line, A C was found to be 160 perches, and B C 2.4 perches; it is required to find the distances to be set off from the random line at every stake left in the line, as also the distance to be set off at 96 perches from the place of beginning,

$$\text{As A C} : \text{A}\,n^1 :: \text{B C} : n^1\, m^1$$

$$160 : 20 \quad :: 2.4 : \quad .3$$

$n^2\, m^2 = .6$; $n^3\, m^3 = .9$; $n^4\, m^4 = 1.2$; $n^5\, m^5 = 1.5$; $n^6\, m^6 = 1.8$; $n^7\, m^7 = 2.1$ perches.

As A C : F A : : C B : F E

160 : 96 : : 2.4 : 1.44 offset at 96 per.

These distances being severally laid off from the random line, to the right or left, according as B is to the right or left of C, will give points in the line A B as required.

2. Being requested to run a line between two neighbors, I ran a random line 146 perches, and found I had missed the true corner by 1.23 perches. What perpendicular offsets must be laid off from this random line, that stakes may be set at every 10 perches from the beginning of the line.

Reckoning from the beginning the several distances will be .08; .17; .25; .34; .42; .50; .58; .67; .76; .84; .93; 1.01; 1.09 and 1.18 perches. In general tenths of a foot will be sufficiently exact.

3. Required the perpendicular distances to be set off from a random line 647 perches in length, terminating at the perpendicular distance of 47.3 feet from the point at which the line in question must end, the stakes in the random line being set at every 40 perches.

Result, 1st dist. 2.9; 2d, 5.8; 3d, 8.8; 4th, 11.7; 5th, 14.6; 6th, 17.5; 7th, 20.5; 8th, 23.4; 9th,

26.3; 10th, 29.2; 11th, 32.2; 12th, 35.1; 13th, 38.0; 14th, 40.9; 15th, 43.9, and 16th, 46.8 feet.

In general it will be most convenient to set the stakes in the true line in a retrograde order, beginning with the last stake in the random line, and returning to the first.

To ensure accuracy in our calculations, we must find the offset corresponding to the distance from the last stake to the end of the line, which being applied to the offset at the last stake in the random line will, if all the calculations are correct, make up the whole offset C B from the true line. Take the last example:—say as 647 : 7 : : 47.3 : 5, which, added to the 16th offset 46.8, gives 47.3 ft.

The following method of finding the offsets may sometimes be used instead of the preceding, or may be used to verify their correctness.

Find the proportional part of the offset for the distance of the last stake from the end of the line, subtract this distance from the whole offset C B, and divide the remainder by the number of equidistant stakes in the line, the quotient will be a number to be continually subtracted from the last offset to obtain the preceding one.

4. Suppose a random line to be run 242 perches, stakes being set in the line 40 perches apart; what distance must be set off at each stake so as to give us the true line, the corner being to the right of the random line 12.1 feet.

First.—As 242 : 2 : : 12.1 : .1 correction for 2p. From 12.1 take .1 the remainder 12 being divided by 6, the number of stakes in the line, gives 2 feet, the difference of the offsets at each succeeding stake.

Offset at the end of the line,	12.1 ft.
" to be deducted for 2 per.	.1
" at the 6th stake,	12.0
difference of offsets,	2.
" at the 5th stake,	10.
	2.
" 4th "	8.
	2.
" 3d "	6.
	2.
" 2d "	4.

Offset at the 2d stake,	4.	
	2.	
" 1st "	2.	
	2.	
" at the beginning,	0.	proof.

The calculation made by the former method is as follows:

As 242 : 40 : : 12.1 : 2. the correction from stake to stake.

Correction at the 1st stake,		2.
"	2d "	4.
"	3d "	6.
"	4th "	8.
"	5th "	10.
"	6th "	12.
"	for 2 per.	.1
"	for 242	12.1

It will be sufficiently exact in many cases to

omit the fractions of a perch in the first and second terms of the proportions, the offsets being small, compared with the line of survey.

5. Run a line 719 perches, when I found I was to the left of the true line 3.7 per. What distance must be subtracted from the 3.7 offset to obtain the offset at 700 perches, and also what number must be successively subtracted from that offset, to give the distance to be set off at stakes set at every 100 perches along the line.

Proportional part for 19 per. is					.1
Offset at the 700 is					3.6
Number to be deducted for 100 per.					
$3.6 \div 7 =$					.514 per.
Offset at the	6th	hundred,	3.08	or	3.1
"	5th	"	2.57	"	2.6
"	4th	"	2.06	"	2.1
"	3d	"	1.54	"	1.5
"	2d	"	1.03	"	1.0
"	1st	"	.52	"	.5

The following rule is that generally given in books on surveying, for determining the true from

a random line: From the given point or place of beginning, run a random line by the given course of the line, and measure the perpendicular distance between the line so run and the sought corner; then,

As the length of the line run,
Is to the said perpendicular distance,
So is 57.3 degrees, or 3438 minutes,
To the difference of variation or correction of the course,

Which, being applied to the given bearing, will give the present bearing of the line.

Then set the instrument at the place of beginning, and run the line by its present bearing.

This method cannot be relied on, as the shortness of the needle, its aberrations, diurnal variation, &c., may lead to error.

The following method of finding the difference of variation is convenient and easily remembered:

To the length of the measured line add its half length, then say,

As that sum is to the perpendicular distance, so is 86° to the correction of the course.

If we have a table of natural tangents, divide the perpendicular distance aforesaid by the length of the line, the quotient is the nat. tang. of the angle of correction; take out the angle corresponding in the table, which will be correct in all cases.

We may use the nat. sines instead of tangs. as far as 5°.

Example 6th. Suppose a line some years ago bore N 40° W 170 per., and that in running by this course, we came out 1.55 per. to the left hand of the true corner: what is the present bearing of the line.

By the first rule:

As 170 : 1.55 : : 3438′ : 31′ correction.

By the second method:

As 255 : 1.55 : : 86° : 31′.

By the third:

1.55÷170=.00912, tang. of 31′.

Hence 40°—31′=N 39° 29′ W, is the present bearing of the line by which it may be retraced by the circumferenter.

Or place the transit at the beginning of the line, and adjust it for observation; set the vernier to

zero, and bring the telescope to bear on a stake in the random line; move the vernier 31′ to the right hand, because the true line is on that side; the telescope will now be in the direction of the line to be run out, which may be done as directed in Proposition 2d.

Rule 4th. Multiply the feet offset by 208, and divide the product by the length of the line in perches, the result will be the correction in minutes.

Rule 5th. Multiply the feet offset by 100, and divide the length of the line in 2 pole chains; to the quotient add its $\frac{1}{25}$ for the correction in minutes.

If a nonius compass be used: Set the nonius 31′ to the right hand, the needle being set to N 40° W, the sights will give the direction of the required line.

This is a great advantage the nonius compass has over the common circumferenter; for having set the nonius, and clamped it to the difference of variation between the present time, and the time a line was formerly run; all the lines of survey run at that time may be retraced, by setting the needle to the given bearings of the lines. This cuts off the possibility of errors arising from applying the *difference of variation* by addition or subtraction to the

several bearings of the lines; that allowance being made by the nonius, it being the same on all the lines of survey, if they have been truly taken.

Example 7th.—In running a line which, some years ago, bore N. 22° 17′ E. 311.7 perches, I found the true corner 4.5 perches to the right hand, what is the present bearing of the line?

Ans. N. 23° 7′ E.

Example 8th.—A line being run by a former course S. 12° 19′ E. 128.7 perches, the corner was found 2.3 perches to the right, what is the present bearing of the line?

Ans. S. 11° 18′ E.

Example 9th.—Being called upon to run the line between two townships, the course and distance of which were given S. $38\frac{3}{4}$° W. 1294 perches, I found, in running by this bearing, that the true corner was on my left 7.12 perches, what is the present bearing of the line between the townships?

Ans. S. 38° 26′ W.

Example 10th.—Wishing to run a line between two points from one of the points I run a course N $89\frac{3}{4}$ E, and measured the distance with a two

pole chain, 129 chains 36 links, when I found the perpendicular distance from the line run, to the point designated, was 17 feet 5 inches to the left hand. Required the course between the required points and the several offsets to be set off from the line run to the true line, the stakes being 50 perches asunder.

Ans. The course is N 89° 31′ E.

1st offset 3 feet 4¼ inches; 2d, 6 feet 8½ inches.
3d, " 10 feet 0½ inch; 4th, 13 feet 4¾ "
5th, " 16 feet 9 inches; for the remaining
part of the line (9.44 perches) 8 inches.

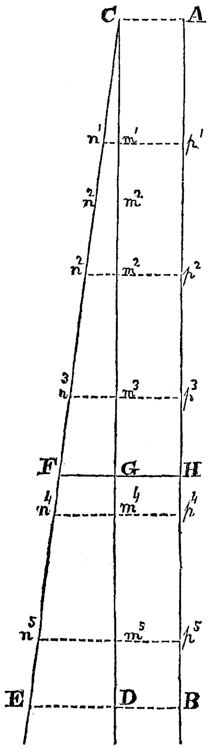
11th. What allowance must be made on a course S 21° E, distance 727 feet, the perpendicular distance from this line to the point required being 17 links.

Ans. Cor. 53′. The course will be S 20° 7′ E.

Case 2*d*. When obstacles prevent running the line directly from the point A, the beginning of the line A B:

Choose a point C near A, so that A C may be perpendicular to A B.

Run the random line C E as before directed, setting equidistant stakes at n^1 n^2 n^3, &c.



Measure E B: from which deduct B D (= A C) the remainder will be E D. Then, by the last case determine the offsets $n^1 m^1$, $n^2 m^2$, &c., to each of which add A C, and we obtain the offsets $n^1 p^1$, $n^2 p^2$, &c., to be laid off from the random line C E to the true A B. If it be required to determine H a point at a given distance from A, we may say; As C E : C F : : E D : F G, to which apply A C (= G H) to obtain F H: whence H is determined. As in the preceding case we may find the angle E C D which applied to the bearing of C E, will give the bearing of C D, that is of A B, because A B is parallel to C D. When E B is less than A C, the corrections $n^1 m^1$, $n^2 m^2$, &c., must be subtracted from A C to obtain the corrections for the points p^1 p^2, &c., in the line A B.

Example 1*st.* Being required to run a line between two points, A and B—I measured a perpendicular A C, equal to 20 feet, and from C run the line C E, S 29° W 130 per., when I found the distance E B was 27 feet to the right of C E; what is the bearing of A B; also what distances must be laid off from C E to determine points in the line A B at 40 per. apart.

Ans. The bearing is S 29° 11′ W.

1st offset, 22. 1 ft. 2d, 24.3 ft. 3d, 26.5 ft.

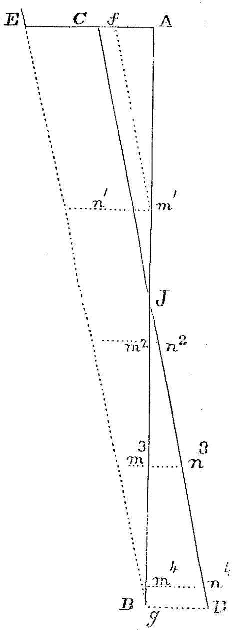
Example 2*d.* Run the random line C E—S 30° W 125 per., and measured A C = 20 ft. and E B = 9 ft., the true line being on the left of the random line. What is the bearing of A B, and what offsets must be measured off to fix points in A B 40 per. apart.

Ans. The bearing is S 30° 18′ W.

1st offset 17.48; 2d, 12.96; and 3d, 9.44 ft.

Case 3*d.* When the random line crosses the true line, between the extreme points.

Let A B be the line required to be run, C D the random line, C and D being on opposite sides of



the line A B. Suppose A B or C D measures 165 perches, A C = 15 feet, and B D = 25 feet. As in the preceding cases, let stakes be driven into the ground in the line C D, equidistant from each other (say 40 perches.) It is evident that the deviation of the random line C D from parallelism with A B is equal to the sum of the distances A C and B D, or 15 + 24 = 40 feet. Therefore, in this case we must take the sum of the distances A C and B D to find the corrections.

$$\text{As } C\,D : C\,n :: A\,C + B\,D : n^1\,m^1 = (A\,C - C\,F)$$

$$165 : 40 :: 15 + 25 : 9.7 = A\,f$$

$$\text{Or As } C\,D : n^5\,B :: A\,C + B\,D : B\,D - n^4\,m^4$$

$$165 : 5 :: 40. : 1.2 = B\,g$$

$$15 \qquad A\,C = 15 \text{ feet.}$$

AF	=	9.7
The 1st offset $n^1\ m^1$	=	5.3
		9.7
2d, do. $n^2\ m^2$	=	4.4
		9.7
3d, do. $n^3\ m^3$	=	14.1
		9.7
4th, do. $n^4\ m^4$	=	23.8
B g		1.2
BD		25 proof.

We cannot subtract 9.7 from 5.3 the first offset, which shows the random and true line intersect between the points, the difference 4.4 is laid off on the other side of the random line, as well as the rest of the offsets to the end of the line. The point I, the intersection of the lines may be found as follows:

As A C + B D : A C : : C D : C I or A I
40 : 15 : : 165 : 61.9

Also A C + B D : B D : : C D : D I, or B I
40 : 25 : : 165 : 103.1

The correction of the bearing is found as in the preceding cases.

40 feet = 2.42 perches.

$2.42 \div 165 = .01467$ nat. tang. of $50\frac{1}{2}'$ this applied to the bearing of C D, will give the bearing of A B. In this case the offsets decrease at the beginning of the line, the correction for the distance between the equidistant stakes, must be subtracted, until the remainder is less than the correction; then subtract this remainder from the said correction, the last remainder will be the distance of the offset at the next stake, to be laid off on the opposite side of the random line to a point in the true one; the random and true line having intersected each other between the stakes, at one of which the offset was laid, on a side of the random line, different from that of the other; after which the offsets are all laid off on this side to the end of the line. Those acquainted with the nature of plus and minus quantities, will readily perceive the reason of all this.

Example 2. Given A B or C D = 125 perches.

A C = 7.7 feet; B D = 6.2 feet required the offsets, the equidistant stakes being 20 perches apart.

Ans. The 1st offset is		5.5 feet.	To the left.
2d,	do.	3.3	
3d,	do.	1.1	
4th,	do.	1.1	To the right of the random line.
5th,	do.	3.4	
6th,	do.	5.6	

The correction of the bearing is 23′.

The lines intersect 69.3 perches from A.

PROPOSITION 5.

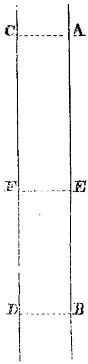
Prolonged Lines.

To determine a point B in the line, A E produced.

Case 1. When the instrument can be placed at E, from which A is visible.

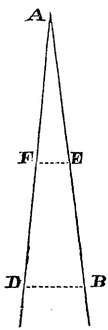
Direct the sights to A, and prolong the line towards B, which continue as far as required.

Case 2. When obstacles prevent A from being seen from E.



Measure off a convenient distance F E, perpendicular to A B; make A C = E F, then project C F to D, a point perpendicular to A B from the point B, and make B D = E F; the point B will be in A E produced.

Case 3. When obstacles prevent A being seen from either E or F.



Run the line A D by proposition 2d. At a point F, measure the perpendicular F E; also measure the distances A E and A B. Then A E : A B : : F E : B D, which being laid off from the line A D, gives the point B in A E produced.

The angle D A B may be found by the rules given in Proposition 4.

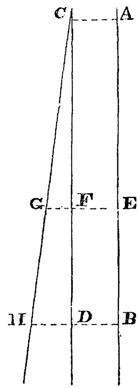
As A E : F E : : 34 38′ : $\angle$ F A E

Or $\frac{3}{2}$ A E : E F : : 86° : $\angle$ E F : F A E

Or F E $\div$ A E = Nat. Tang. $\angle$ F A E.

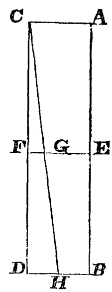
Place the instrument at A and make the ∠ F A E as found above; then run out A E B.

Case 4. When it is impracticable to run from A,



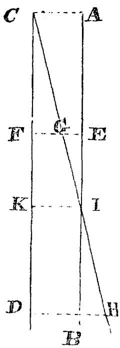
Set the instrument at C, a point perpendicular to A B, opposite to A. Run a line C G H to H, opposite to B. Measure G E, from which deduct A C = F E, the remainder is G F. Then, as in the preceding case, as A E : A B : : G F : H D, to which apply B D = A C, we get H D + A C = H B.

Case 5. When G E is less than A C.



From A C deduct G E, the remainder is F G. Then, G C : C H : : F G : D H and H B = B D—D H = A C—D H. So the point B is determined.

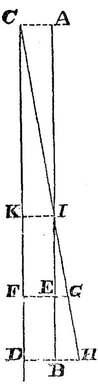
Case 6. When the random line C H crosses the line A E prolonged.



In this case, F G = A C—G E
C G : C H : : F G : D H.

D H being greater than A C, their difference is B H, which must be laid off to the contrary side of C H, to which A C was laid; that is, A and B are on opposite sides of C H, as D H : A C : : C H : C I. The point of intersection of the random and true lines may, therefore, be exactly designated.

Case 7. When the random line crosses the true, between A and E.



In this case, G E will be on the opposite side of the line A E, to which A C is; therefore,

F G = F E + E G = A C + E G

As C G : C H : : F G : D H

H B = D H — D B = D H — A C

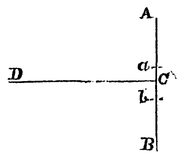
A and B are on contrary sides of C H. I, the point of intersection is found as in Case 6. The > D C H is found by Case 3.

PROPOSITION 6.

To retrace the lines of a Survey.

Case 1. When the angular points are established, run the lines from one angular point to another, by the methods already given.

Case 2. When the angular point is known to be in a right line with two other points, and at a given distance from each of them.



Let A and B be the given points, C the point in a right line between them, which is to be determined in order to trace the line D C.

Run the line A B as before directed, from A towards C, measure the distance the point C is known to be from A; also, from B to C, measure the distance C is known to be from B, then, if these distances both terminate at C, the point is determined. The line C D may then be run out by the methods already given. But if the measures from A and B do not terminate at the same point C, which in practice will often be the case, measure the given distances from A and B towards C; the distance measured from A, terminating at

a; that from B at *b*. Then it will be, as the given distance of A from B, is to the given distance A C; so is the distance *a b*, to the correction *a* C, or as A B : B C : : *a b* : *b* C the correction on B *b*. The correction *a* C applied to A *a* or *b* C applied to B *b*, will determine the point C.

This method is also applied, when there is not full measure.

Example. *Given* A C = 75 perches; B C = 150 perches, but in measuring these distances, there is found an excess of measure, *a b* = .6 perches, required the corrections. As 75 + 150 : 75 : : .6 : *a* C = .2; or 225 : 150 : : .6 : *b* B = .4.

Therefore, A C = A *a* + *a* C = 75 + .2 = 75.2 perches, B C = B *b* + *b* C = 150 + .4 = 150.4.

Example 2*d*. *Given* A C = 325 perches, B C = 175, and *a b* = 1.5 perches, excess measure, what are the true distances?

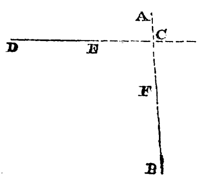
A C = 325.975 B C = 175.525.

Example 3*d*. *Given* A C = 120 chains; B C = 80 chains, and a deficiency of full measure, *a b* = 20 links. What is the lengths of A C and B C by this measure?

Ans. A C = 119.88 chains, and B C = 79.92 chains.

Case 3. When the adjacent lands A C D and B C D have been surveyed at very different periods of time, measure several lines of the land adjacent to A C D, and say as the sum of the lines measured, is to the line A C; so is the gain or loss of measure on the lines measured, to the gain or loss of measure to be applied to A C. In like manner find a correction of the measure of B C, by measuring several lines of the adjacent land B C D. Use these corrected distances for A *a* and B *b* in the preceding case.

Case 4. When there are line marks in the line D C at D and E, and also in the line A B at B and F.



Prolong the line B F towards A, by the method already described; also prolong D E until it intersects the prolonged line B F in C, which will be the point sought.

Case 5. When there are marks only at D and B, runs out ome of the lines on the adjacent lands,

which are nearly parallel to the lines to be run, by which the difference of variation is obtained, which being applied to the former bearings, gives the present bearings of the lines. The lines may then be run out. This may be done with both adjacent tracts of land, and a mean of the results taken.

A reason for selecting lines nearly parallel to the line to be run, is that the difference of bearings of a line as shown by two compasses, will be the same on lines nearly parallel to it. When this difference is applied to lines at nearly right angles with it, a considerable difference will very often be found, which frequently leads to error, unless carefully guarded against.

Case 6. When there is given the bearings and distance of the line A B, running from the point A, and the year in which it was run.

The only method likely in this case to approach a satisfactory result, is the following:

If there have been a line or lines in the neighborhood, run the same year, (or thereabouts) go to the premises and run them out, by which you get their present bearing, and therefore the difference of variation between the present

time, and the time at which the surveys had been made; this difference allowed on the bearing of the line to be run out, will give its present bearing by which to run it out.

If, however, no survey had been made of any lands in the neighborhood, by which the difference of magnetic variation may be found; then, in such case, if the annual rate of increase or decrease in the magnetic declination be satisfactorily known, we may ascertain the change of variation in the interval of time which applied to the given bearing of the line to be run, we shall have its present bearing. The variation at West Chester, in 1845, is 4° 2′ W.

The variation of the magnetic needle in declination, is subject to much irregularity, in some instances increasing, in other decreasing, and some years having scarcely a perceptible motion. The annual variation at Philadelphia, has been stated at 1° in twenty years. In the neighborhood of West Chester, it is about 1° in sixteen years, in Warminster Township, Bucks County, 1° 3′ in fourteen and one-third years. At any place there

is much irregularity in a lapse of years. It must, therefore, be a matter of uncertainty whether we have the correct bearing of the line, even when the change for years has been ascertained with the utmost care.

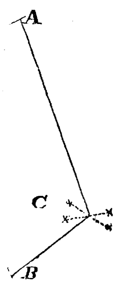
Another source of error in this case, the diurnal variation, may be properly mentioned here. If a survey be commenced early in the morning, which is not completed until one or two o'clock, P. M., of a very warm day, it will be found that the bearing of the first line of survey will vary several minutes, sometimes a quarter of a degree from its bearing in the morning. In the winter season, this difference will seldom exceed five minutes of a degree, but in very warm weather it may amount to fifteen minutes. There will be little difference in cloudy weather. Surveys should, therefore, as far as practicable, be made in the cool part of the day. A line which is to be established from the course only, should be re-run at nearly the same season of the year, a day chosen of much the same temperature, and the same time of day, in order to ensure the nearest approach to accuracy the case will admit of.

Other sources of error are the eccentricity of the compasses used, the difference of polarity or direction of the needles used, &c., all which should be carefully guarded against. If the surveyor when running old lines were to note the difference between the bearing now found and that given, by applying this difference to the variation of his needle, he may determine very nearly the magnetic variation at the time of the former survey.

A collection of observations of this kind would enable him to ascertain the rate of increase or decrease of the variation of the magnetic needle; and would be highly valued by those who may be investigating this perplexing subject.

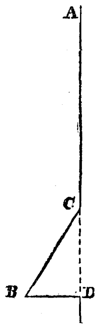
Case 7. When the angular point C is to be determined from the distances A C and B C only, the points A and B being known.

Measure A C as nearly as may be in the direction of C, and at the end of the distance, set two stakes a few feet apart, so that a line joining them may be at right angles with A C. Also, measure B C, and set two stakes a few feet apart at right angles to B C. Then



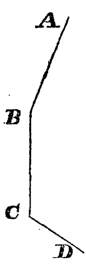
a line joining the former two stakes will intersect a line joining the latter two in the point C, the angular point required.

Case 8. When C B bears but a few degrees from A C.



Prolong A C towards D. With the angle of deflection B C D as a course, and B C a distance, enter the traverse table, and take out the latitude C D, and make D B perpendicular thereto, and equal to the departure; the point C and the line B C are determined by making D C = the latitude from the traverse table.

Case 9. When local attraction affects the needle on the several lines.



Set the instrument at A, direct a stake to be set at B, let the needle down on its centre pin, and direct the sights to B. If the needle does not show the same bearing as formerly, move the vernier plate till it does. Remove to B, direct the sights to A. If the needle does not show the same course for B A as at A, there is local attraction. Move the vernier till the needle

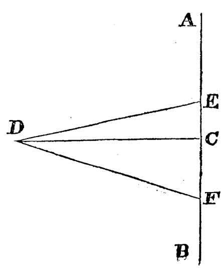
shows the reverse of the given bearing of A B. The needle set to the given bearing of B C will give its direction. Next remove to C, and take a sight to B, move the vernier until the needle gives the reverse bearing of B C. The needle set to the bearing of C D will give its direction; and so proceed. Otherwise, apply the angle A B C to the bearing of A B, to get the bearing of B C; apply the angle B C D to get the bearing of C D, &c. This may be very accurately done with a theodolite or transit.

PROPOSITION 7.

On Distances, &c.

Case 1. Through a given point to run a line D C at right angles with a given line A B.

In the line A B choose two stations, E and F such that the angle E D F may be less than 10°, the point C falling between E and F.



With an instrument measuring angles to minutes, mea-

sure the angles E F D and F E D, the complements of which give C D E and C D F; also, measure E F; then, as the sum of the angles C D E and C D F in degrees and minutes, is to either of them, as C D F in the same measure, so is the base E F, to the part C F of the base, corresponding to the < F D C, from which the point C is determined.

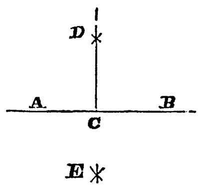
If ∠ C D E be used in the above proportion, we get E C; or correctly, as the sum of the nat. tangs. of the angles C D E and C D F, is to the nat. tang. of either of them, so is E F the sum of the segments E C and C F, to the segment corresponding to the angle used in the second term of the proportion.

Otherwise, subtract the bearing of A B from 90°, the remainder changing N. to S. or S. to N., is the bearing of C D; then, by trials make C D this course; and C will be the point required.

Case 2. From the point C in a right line A B, to trace a line C D at right angles with it.

This may be readily done, any of the instruments used in taking angles, or with the cross

mentioned in the choice of instruments. It may also be done with the chain as follows:



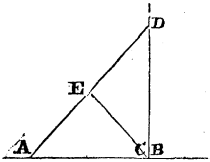
Make A C = B C = 20, 30, or any other number of links less than a chain, place one end of the chain at A, and with the other end, trace an arc on the ground; remove the end of the chain from A to B; with the other describe a second arc, cutting the former in D. A line joining D C will be at right angles to A B. In the same manner, another point E on the other side of A B may be found.

Or thus, set the compass at C, and take the bearing of A B. Subtract this from 90°, changing N to S or S to N, gives the bearing of C D.

Another method: make A C = 4; with A as a centre and a radius 5 describe an arc; with the centre C and radius 3 describe an arc, cutting the former in D the point required.

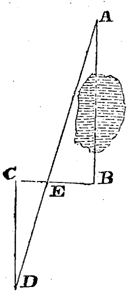
Any multiples of 3, 4, 5, as 6, 8, 10, or 30, 40, 50, &c., will form right ∟d triangles.

Third method: place one end of the chain at C, the point at which a right angle is to be made, ex-



tend the other end to any convenient point E; with E as a centre, move the other end from C (the chain being radius,) until it crosses the line A B in A; prolong A E towards D, making D E equal to A E. Join C D which will be at right angles with A B.

Case 3. To measure an inaccessible distance A B.



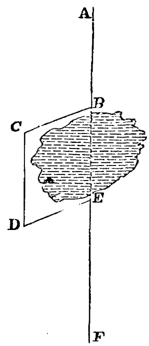
Make B C at right angles to A B, at C any convenient point. Make C D at right angles to C B, or parallel to A B. Set a staff at E in the line C B, and in a range with A D. Measure B E, E C, and C D; then as E C : B E : : C D : A B.

This proportion will hold good, if B C make any angle whatever with the parallel lines A B and C D.

Note.—If B C = C E, then A B = C D.

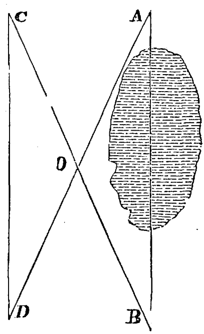
Case 4. To determine the distance A F, which cannot be directly measured, on account of the obstacle between B and E.

Trace the line to B, run B C. From C run C D parallel to A B. From D run D E parallel to B C, and make it equal to it. Then run E F the same course as A B: F will be a point in the line A B continued. The distance A F will be equal to the sum of the distances A B, C D and E F.



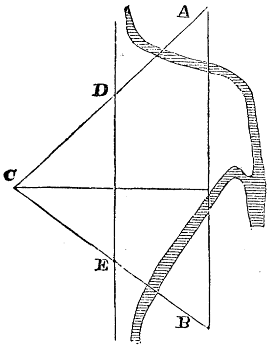
Case 5. To find the bearing and distance of A from B, accessible only at its extremities.

Choose a point O from which A and B are both accessible. Prolong A O to D, making D O equal to A O; also, prolong B O to C, making O C = O B. Join C D, which will be equal and parallel to A B. Its bearing and distance is therefore determined.



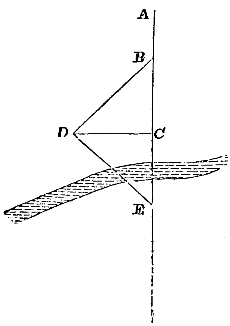
Case 5. When A and B are inaccessible.

Plant a staff at C and find the distance A C and C B by the preceding methods:



Make C D as many parts of A C as you do C E of C B; join D E and measure it. Then, as C D : C A : : D E : A B, or as C E : C B : : D E : A B D E is parallel to A B.

Case 6. To prolong the line A B across a river, &c., and determine the distance across to E.



At a point C make the angle B C D a right one, at D make the angle B D E a right angle. Measure B C and D B. Then B C : B D : : B D : B E. If D C be measured, B C : D C : : D C : C E.

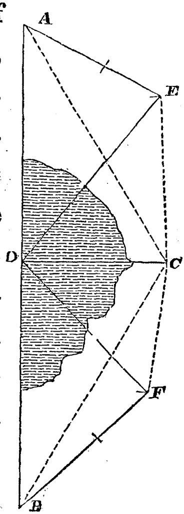
Otherwise. Make $\angle$ C D E $= 45°$, then C D will be equal to C E, the breadth of the river.

If $< CDE = 26° 34'$ then $CE = \frac{1}{2} CD$
" $< CDE = 33° 41'$ " $CE = \frac{2}{3} CD$
" $< CDE = 45°$ " $CE = CD$
" $< CDE = 56° 19'$ " $CE = \frac{3}{2} CD$
" $< CDE = 63° 26'$ " $CE = 2 CD$

Or, if $CBD = BDE = 60°$, the triangle B D E becomes equilateral, and $BD = BE$.

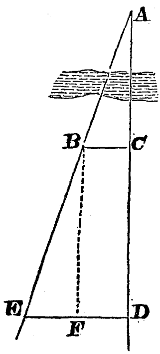
Case 7th. To find the horizontal distance A B through a precipice or clift too steep to be measured with the chain.

Plant a staff at C on the top of the cliff, aligning it with A and B, both of which may be seen from C. Run the line A E and measure it. Also measure the angles $CAE = DAE$; and $CEA = DEA$ (the point D being directly below C.) Find, by trigonometry, the distance A D. In like manner from B run B F and measure it, also the angles $CBF = DBF$; and $BFC = BFD$ to find B D. Then A B is the sum of the hori-



zontal distances A D and B D. In this solution the planes A D C, E D C, B D C and F D C are conceived to be at right angles to the plane A E D F B. By taking the altitude to the top of the staff C from two stations on each side of the steep, the horizontal distance may be found.

Case 8. To measure an inaccessible distance A C.



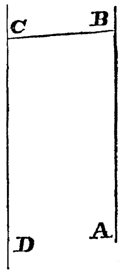
Plant a staff at D in a line with A C. Run B C and measure it; and make E D parallel thereto, and measure E D and also C D. Then E F (= E D — B C) : B C : : C D to A C. This case is applicable when B C and E D differ considerably, that is, E F bears a due proportion to B F or C D.

Note.—This Case should have followed Case 3.

PROPOSITION 8.

Parallel Lines.

Case 1. From a given point C, to run a line C D parallel to A B.

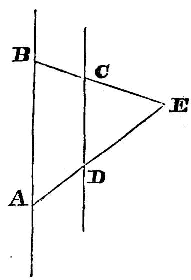


Run the line A B to find its bearing, remove the instrument to C and set the needle to this bearing, the sights will then be parallel to A B, so C D may be run out.

Or set the instrument at B and measure the angle A B C. Remove to C, making the angle B C D equal to the supplement of A B C. Then will C D be parallel to A B.

If a transit be used after having removed to C, reverse the telescope on its axis, bring it to bear on B (the vernier being at ∠ A B C,) clamp the lower plate, bring the vernier to zero, the telescope will be parallel to A B; reverse the telescope on its axis, and set a stake at D.

Case 2. When B is not accessible from C,

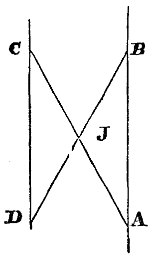


Plant stakes at A, B, C, also at E, in a line with B C. Find, by the preceding methods, the distances A E, E B and E C.

As E B : E C : : A E : E D. This being laid off in A E to D, a line joining C D will be parallel to A B.

7

Second Method.—Run A C and make $\angle$ A C D = B A C.



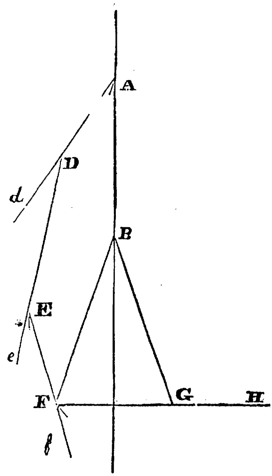
Otherwise. Bisect A C in I. In B I produced, make I D = B I. Join C D, which will be parallel to A B.

PROPOSITION 9.

To determine a point C in a right line with A B, the point B being a steeple, or other conspicuous object, which is visible from A and C, and inaccessible from both.

Measure the angles of deflection B A D *d* D E, *e* E F, D E and F being suitable points for measuring the angles of deflection. The station F being selected as near C as can be judged upon the ground.

The angles of deflection at D and E being to the left, their sum must be diminished by the



angle at A: the remainder is the deviation from parallelism of the lines A B and E F. This remainder subtracted from 90°, or a right angle gives the angle of deflection *f* F G at F, to make F G at right angles with A B prolonged. Run F C G, measuring the distance F G. Observe the angles B F G and F G B, the complements of which are F B C and G B C. The sum of these complements is F B G.

As the angle F B G, in degrees or minutes,
Is to the angle F B C in the same measure,
So is F G
To F C,
Or, as $\angle$ F B G : $\angle$ G B C : G F : C G.

From either of these proportions the point C is determined, which will be in A B produced, F G is the complementary course of A B. It may be

run out where there is no local disturbance of the needle by setting the instrument at F, and run F G by this complementary course. The base F G of the triangle B F G should be of such a length, that the angle F B G may be only a few degrees; if it should be too great, the distances F C and C G will not increase in the same ratio with the angles F B C and C B G.

N. B.—If F B G is greater than 10°, the segments F C and G C should be found by the usual rules of Trigonometry.

The following proportion is in substance the same as the preceding. From the external angle B G H, take the $<$ B F G; then say,

As this remainder is to the difference between the angle B F G and a right one; so is F G to F C as before.

It may sometimes be convenient to take several stations in deflecting from A to F; but in all cases the angles of deflection to the right hand must be added together, and also those on the left of the lines deflected; the difference of these sums will be

the deviation, from parallelism, of the last line deflected from the line A B.

If the deflected angles on the right exceed those to the left, the difference must be laid off to the left, and vice versa: the telescope will then be parallel to A B.

If at every station we arrive at, we set the vernier to the same degrees as at the last station, reversing the telescope on its axis or in its wyes, and bringing it to bear on the last station point; then, having clamped the lower plate to the tripod, bring the telescope in its direct position on the next station, the vernier will perform the additions and subtractions of the angles of deflection; consequently, when we arrive at any station when the instrument is adjusted by the last station point, bring the vernier to zero, the telescope will be parallel to A B—but if set to 90°, it will be at right angles with it. We may make any angle whatever with A B, by setting the vernier by means of the tangent screw to that angle.

Given $<$ F B C $= 2° 5'$; G B C $= 2° 55'$ and

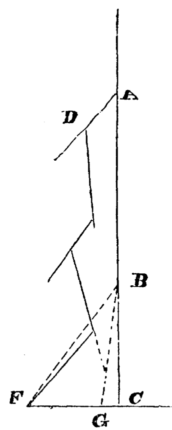
F G = 12 ft., to find F C (the points F and G being determined as above).

As F B G (300′) : F B C (125′) : : F G (12 ft.) : F C = 5 ft.

F B G (300′) : C B G (175′) : : F G (12 ft.) : G C = 7 ft.

Case 2d. When C falls without the triangle F B G.

Find, as before, the angle F B G, the angle G B C, which is equal to the difference between B G C and a right angle, and the distance F G.



As F B G : < G B C : : F G : G C, which, being laid off on F G produced, determines C, a point in A B prolonged. The contents of this proposition and Case 1st of Prop. 7th, are believed to be new; nothing of a similar character is to be found in any publication with which I am acquainted.

Given F B C = 50′; G B C=10′ and F G = 8 ft.

to find G C; F G being determined as above directed.

As $\angle$ F B G ($= 50' - 10' = 40'$) : G B C ($10'$) : : F G = (8 ft.) : G C = 2 ft.

Given F B C = $83'$; G B C = $17'$ and F G = 33 ft. to find G C. *Ans.* G C = 8.5 ft. if G is between F and C; but if not; G C = 5.61 ft.

PROPOSITION 10.

Dividing Land.

An easy rule for finding the angles of a right angled triangle, the sides being given. To the hypothenuse add half the longer leg. Then, as that sum is to the shorter leg, so is 86° to the angle opposite the shorter leg. This rule, which is easily remembered, is very useful in many calculations in the field, where tables cannot be conveniently used. The greatest error does not exceed—4 minutes. The rule is therefore sufficiently exact for most purposes in surveying to which it may be applied.

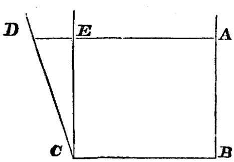
Example. Given the 3 sides of a right angled triangle, 30, 40 and 50, to find the angles:

(50 + 20) : 30 : : 86°—36° 53′ the less angle.

Example 2d. Given the hyp. and greater leg of a right angled triangle 50 and 40 angle opposite less leg $36\frac{6}{7}$°, to find the less leg.

As 86° : $36\frac{6}{7}$° : 70 : 30, as required.

This rule may be readily applied in cutting off a trapezoid from a given tract of land that shall contain a given number of acres, the angles being nearly right angles.



Let there be given A B south, B C west 40 per. and C D N. 4° W., to cut off a trapezoid A B C D containing 5 acres, by a line A D parallel to B C.

First, 800 ÷ 40 = 20 = C E approximate.

In this case the leg and hypothenuse are nearly equal, we may use one for the other, therefore,

As 86° : 4° : : (20 + 10) : 1.4 = E D approximate.

A D = A E + E D = B C + E D = 40 + 1.4 = 41.4 approximate.

Twice the area of a trapezoid, divided by the sum of the parallel sides, gives the perpendicular distance between them.

$1600 \div (40 + 41.4) = 1600 \div 81.4 = 19.66 =$ C E = A B.

As 86° : 4° : : (19.66 + 9.83) : 1.37 = D E.

A D = 40 + 1.37 = 41.37 sufficiently correct.

$1600 \div (40 + 41.37) = 19.66$ as before (the proof.)

The angle B being a right angle, no correction is required on that side of the trapezoid.

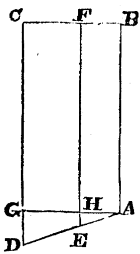
Another method is to find the area of the triangle D C E, and cut off a small trapezoid equal to it, either within or without, as the case may require. In the preceding example E C + $\frac{1}{2}$ D E = area of C D E = 20 + .7 = 14., this, divided by A D 41.4, gives .34, and this subtracted from E C, 20, because A D is greater than C B, gives the correct value of E C = 20 — .34 = 19.66, as before.

In most cases the use of the traverse table is more expeditious to find D E. Taking the approximate value of C E in a lat. column, under 4°,

gives, in a departure column, 1.4 = D E, from which a correct value of C E is found. With 19.66 in a lat. column, under 4° in a distance column, is found 19.71 = C D.

Example 2d. *Given* A B south, B C west 40 per., and C D N. 4° E. to find A B, C D and A D, when the trapezoid A B C D contains 5 acres.

Ans. A B = 20.36 perches.
C D = 20.41 "
A D = 38.58 "



When it is required to cut off a trapezoid from a trapezoidal piece of land, it may sometimes be done in the following manner:

Given A B N. 40° E. 44.4 perches, B C S. $50\frac{1}{2}$ E. 60.8 perches, C D S. 40° W. 46 perches, and D A N. 49° W. 60.9 perches, to cut off 5 acres by a line E F parallel to A B.

First.—800 ÷ 44.4 = 18 = E A approximate.

D g = D C — A B = 46 — 44.4 = 1.6.

A D : A E : : D g : E h or 60.9 : 18 : : 1.6 : .47.

E F = F h + h E = A B + E h = 44.4 + .47 = 44.87.

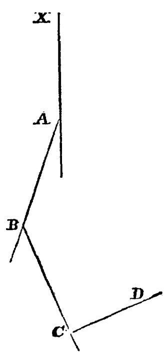
A h = F B = 1600 ÷ (44.4 + 44.87) = 17.93 the perpendicular corrected.

In a lat. column with 17.93 under $1\frac{1}{2}° = \angle g$ A D in a distance column, is found 17.94 = A E.

When calculations are made by J. Gummere's rule, (the sides A D and B C, as in the above example, being nearly parallel,) great care must be used in extending the decimals to 3 places to ensure accuracy, as two decimals only may throw the line E F a pole from its true position in many cases that occur in practice.

PROPOSITION 11.

To determine the correct bearings of the lines of survey where local attraction deflects the needle from its usual direction or magnetic position.



Let A B C D be several sides of a survey on which the needle is disturbed by some extraneous matter.

With the compass or circumferenter: Place the the compass at A, take a back sight to X, the last station, and note the bearing, then sight to B and note its bearing. Having the bearing of A X and A B, both from the same station A, we can find the angle X A B as correctly as if the needle settled in its true position, for the needle must be equally affected when the bearings were noted. Remove to B, and take a back sight to A, noting its bearing, then direct the sights to C, and note its bearing, from which the angle A B C will be correctly obtained. Thus proceed until all the angles are taken. If the entire survey has been made as above directed, the sum of all the internal angles will be equal to twice as many right angles as the figure has sides, diminished by four right angles. If this sum, as in practice will be likely to be the case, should differ a few minutes from what it should be, the minutes of error may be distributed among the angles by addition or subtraction, according as there is defect or excess in the sum of the observed angles.

Now, having all the correct angles, assume some

side of survey as X A to be correct, being least affected by local attraction; then applying the angles severally as they come in order of survey, we will have the bearings of the sides as correct, relatively, as if no local attraction existed.

If a nonius compass be used, place it at A, take the bearing of A B, remove to B, take a back sight to A and clamp the sights upon it. Unclamp the nonius plate, and with the pinion and rack move the nonius plate until the needle gives the reverse bearing of A B, which it had at A. Unclamp the sights and bring them to bear on C, the needle will show the correct relative bearing of B C, which note. Remove to C, take a back sight to B, and clamp the sights upon it; move the nonius plate by the rack until the needle shows the reverse bearing of B C; unclamp the sights and take the bearing to the next station, and so proceed till the survey is completed. The relative bearings thus obtained will be as correct as if no local attraction influenced the needle.

If a theodolite or transit be used, the internal angles may all be measured by the limb of the in-

strument, without regard to the needle. From which, having also the bearing of one line, the bearings of all the lines may be found.

The external angles, or angles of deflection, may also be taken as follows:

Place the instrument over A, reverse the telescope on its axis or in its wyes, set the vernier to zero, and bring the sight to bear on X; then clamp the lower or graduated plate in this position, reverse the telescope to bring it again in its direct position, bring telescope to bear on B, (by means of the tangent screw or rack,) the index or vernier will, being read, give the angle of deflection of the lines X A and A B. Remove to B and take the angle of deflection to C in the same manner as from A to B; proceed thus the entire circuit of the survey. If all the angles of deflection have been outward, their sum must, if correctly taken, be equal to four right angles, or 360°. If any of the angles are re-entering, the sum of the external diminished by the sum of the re-entering angles, will be equal to 360°.

If the telescope be reversed, the vernier being at

the same division as at the last station, the hair of the telescope cutting the last station point, then clamping the lower plate to the tripod, reverse the telescope to bring it in a direct position, bring it to bear with the tangent screw on next forward station, the vernier will show the angle of deflection. Proceeding thus from station to station, the vernier will give the sum of the angles of deflection, and hence, when we arrive at the first station, the vernier will be at zero, or the point at which it was placed when the survey was begun. Its distance from this point is the sum of the errors in observing the angles which may be distributed amongst them, so as to make the proper sum or quantity for the angles.

If there should be a considerable difference or error it would be advisable to retrace some of the lines until the error be discovered.

CALCULATIONS.

It is the practice with some surveyors to read the bearings of lines to quarter degrees, and note

the distances in chains and links, which, in calculations, they reduce to perches and hundredths.

In measuring distances, where the line to be measured is one hundred perches or upwards in length, it must be evident to those who may be acquainted with the ordinary mode of measuring, that that distance cannot be measured to the hundredth of a perch, and frequently not even to the tenth.

Again, when courses are read to the nearest quarter degree, there is a probability of an error which may reach to half that quantity or $7\frac{1}{2}'$, which, in the distance of 100 perches, gives 2 tenths of a perch departure from the point of termination. Therefore, when bearings are read to quarters of a degree, and distances measured to tenths of a perch, it will not conduce to accuracy to extend the calculations for the area to hundredths or another decimal figure.

The bearings of lines should always be read to the nearest five minutes, and distances over 100 perches need not be more exactly noted than to tenths of a perch.

In laying out town lots, or where the utmost precision is desired, angles or bearings should be measured to the nearest minute, and distances to hundredths of a perch, or tenths, or hundredths of a foot.

For this purpose a theodolite or transit should be used to measure the angles, and a twenty feet frame, with a level or plummet attached, having a slider affixed at either end of the frame, to adjust it to horizontal admeasurement. To measure a line with the frame, in the first place, the line should be "boned," as it is technically termed, that is, pegs or short stakes, at the distance of 20 feet, should be driven in the line nearly even with the surface of the ground; then placing one end of the frame at the beginning of the line, the other on the first stake, after having adjusted the frame to a level, make a fine scribe or mark on the top of the stake, precisely at the end of the *frame;* next, bring the *frame* forward, adjust the hind end to the scribe on the stake, bring it to a level and scribe the second stake at the end of the frame; and so proceed to the end of the line.

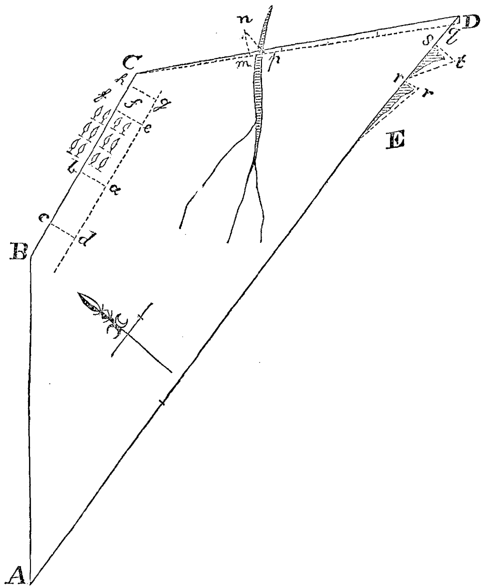
A line, several hundred feet in length, if measured as above directed, will be within an inch or two of the truth.

METHOD OF OBTAINING THE FIELD NOTES OF A TRACT OF LAND ACCURATELY.

Having given directions in the preceding part of this work, best suited to determine correctly the position of any line of a survey which may be desired to be run out or retraced, it may be proper here to give an example embracing a number of those cases, which occur in practice, so as to exhibit the application of the rules which have been given.

Let us suppose it be required to survey a tract of land A B C D E A with a transit and common two pole chain.

Place the instrument, by means of a plummet, exactly over the point A of beginning. After having adjusted it for observation, set the vernier to zero, and clamp it there; next, let the needle down upon the centre-pin, revolve the instrument



so as to bring the needle to the north and south points of the compass box, clamp the lower plate to the staff head, unclamp the vernier plate and bring the telescope to bear on a staff set at B, the second station; the course may be read from either the vernier or needle, or both, which is preferable. Suppose the vernier reads 50° 21′ and the needle N.

50° 20′ E., also A B measures 68 chains and $47\frac{1}{2}$ links. The course and distance may be set down A B N. 50° 21′ E. 137.9 perches, using the course shown by the vernier in preference to that shown by the needle. The observations at A having been finished, remove the instrument to B, adjust it for observation; reverse the telescope, the vernier being at 50° 21′, the bearing at the last station A B, bring the telescope to bear on a pole at A, clamp the instrument to the tripod, the zero points or north and south points of the graduated plate and the compass box will be in the magnetic meridian, and consequently parallel to its position at A; the vernier will therefore show the bearing of a line, the same as the needle. Unclamp the vernier plate and bring the telescope to bear on a third station C. Let the needle settle—suppose it reads N. 79° 55′ E; at the same time the vernier reads 79° 54′. In measuring this line B C, an obstacle, a clump of bushes and swamp prevents its being directly measured further than *b*, 20 chains from B. An offset, *b a*, being measured 12 feet at right angles with B C, also at *c*, a point between

B and b, a perpendicular offset of 12 feet was made from c to d. Prolong the line $d\ a$ to e, where set off an offset $e\ f = 12$ feet; continue the line $a\ e$ to g, make $g\ h = 12$ feet, perpendicular to $a\ e$, continue $f\ h$ to C. It is found $a\ e$ measured 14 chains 25 links, and $f\ c = 10$ chains 21 links, so the whole line B C measures 44 chains 46 links; the bearing and distance of B C is N. 79° 54′ E. 89.8 perches.

Place the instrument at C, take a back sight to B, the vernier being at 79° 54′, clamp the lower plate to the tripod, release the upper plate. The station D being out of view, run a random line in that direction, as nearly as may be, setting the vernier to 130°; the needle reading S. 50° E. at the same time, run the line 47 chains to the bank of a deep creek at m, on the opposite side of which set a stake at p, a point in the continuation of the random line C q. Measure a perpendicular $m\ n = 6$ chains, and make the angle $m\ n\ p = 26° 34′$; consequently $m\ p$ is equal the half of $m\ n = 3$ chains. From p continue the random line to q, 19 chains further; when we arrive opposite to D

on the left of the random line, D C is 69 chains = 138 perches. The corner D from q is 9.25 feet = 56 perches; $\frac{3}{2}$ C D = 207 : .56 : : 86° : 14′ the correction which apply to C q, gives the bearing of C D 129° 46′ or S. 50° 14′ E. 138 perches. Stakes being set in the random line C q at every 20 perches, the calculation for the offsets is as follows: 138 : 20 : : 9.25 ft. : 1.35 ft. The distances to be laid off at the several stakes will be 1.35; 2.7; 4.05; 5.4; 6.75 and 8.1 feet, these distances having been laid off, establishes the line C D.

At a point in the random line C q, near q, and in the line D E let the instrument be set for observing the course of D E. The telescope being reversed and brought to bear on a back stake p, in the random line C q, the vernier at the same time reading 130°, let the lower plate be clamped to the tripod, bring the telescope to its forward direction, and by means of the tangent screw, make the hair cut E or a stake in a range with D E, the vernier gives for the bearing of D E 270°; the needle due west measuring 7 chains 25 links from D towards E, a high and steep rock was

encountered, on the top of which a stake was set at *r* in a right line with D E. At the termination *s* of the 7 chains 25 links, a right angle *s t* was set off from the line D E, equal to 5 chains, making the angle $s\ t\ r = 63°\ 26'$, the distance *r s* must be 10 chains. From *r* to E the ground was a regular slope. I set my instrument at *r*, and took the depression to E 11° measuring the oblique line *r* E 17 chains 25 links. With 11° as a course and $17\frac{1}{2}$ in a distance, the latitude is 16.68 or 16 chains 34 links, equal to 33.36 perches for the horizontal distance; therefore D E measures, horizontally, 34 chains 9 links or 68.36 perches. The instrument being removed to E and adjusted as at the other stations, the telescope being directed to A, the vernier will read 266° 12′; the needle S. 86° 10′ W., and measuring the distance A E, it will be found to be 116 chains 20 links. The instrument next placed at A, reverse the telescope, and bring it to bear on E, the vernier being at 266° 12′, and clamp the lower plate to the tripod. The telescope being in its direct position, and brought to bear on B, the vernier, if the work is correctly done, will read

50° 21′; this being the point at which the vernier was set in first setting out, is the proof that the angles have been correctly measured. The courses and distances in this example will be as follows:

A B	N. 50° 21′ E.,	distance	137.9 perches.
B C	N. 79° 54′ E.,	"	89.8 "
C D	S. 50° 14′ E.,	"	138.0 "
D E	West,	"	68.36 "
E A	S. 86° 12′ W.,	"	232.8 "

The outward angles by the vernier will be at

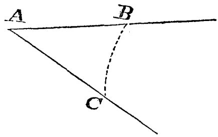
A	50° 21′,	or 144° 09′	< of deflection.
B	79° 54′,	or 29° 33′	"
C	129° 26′,	or 49° 32′	"
D	270.	or 140° 34′	"
E	266° 12′,	or 3° 48′	"

Sum of right hand <'s 363° 48′ positive <'s.
" left " " 3° 48′ negative "

Proof, 360° 00′

Note.—It is sometimes conducive to accuracy to measure diagonal lines, or lines to opposite corners of the tract surveyed.

N. B. Angles of a survey may be measured with the chain as follows:



Let A be the angular point, A B the direction of one of the lines, and A C the line of direction of the other line. Measure A B and make A C equal to it, and join B C and measure it. The < A in the isosceles triangle A B C is readily found.

Or, A B, B C and A C may all be unequal in which case the angles must be found by the rules for solving oblique angled triangles.

It sometimes happens that an old road is required to be straightened between two given points, in which case it may be desirable to know approximately the course between the given points. This, if the courses do not differ much amongst themselves on the old road, may be ascertained as in the equation of payments, using the courses and distances as the payments and time are used in the ordinary rules of arithmetic.

Given N. 41° E.	20 per.	$41° \times 20 =$	820
N. 43° E.	30 "	$43° \times 30 =$	1290
N. 42° E.	80 "	$42° \times 80 =$	3360
N. 44° E.	120 "	$44° \times 120 =$	5280
N. 40° E.	200 "	$40° \times 200 =$	8000

What is the course of the straight line joining the extreme points?

$$450)\ 18750$$

$$41\tfrac{2}{3}°$$

The course is N. $41\frac{2}{3}°$ E., nearly.

Or thus, taking the several courses from 41°.

$$0° \times 20 = 0$$
$$2° \times 30 = 60$$
$$1° \times 80 = 80$$
$$3° \times 120 = 360$$
$$1° \times 200 = 200$$

$$450)\ 300$$

$$\tfrac{2}{3}°$$
$$41$$

N. $41\frac{2}{3}$ E. as before.

By the traverse N. 41° 35′ E.

Given	N. 2° E.	60	perches.
	N. 3° W.	90	"
	N. 1° W.	80	"
	N. 1° E.	120	"

to find the course of the strait line joining the extreme points.

					E.	W.
N. 2° E.	×	60	=		120	—
N. 3° W.	×	90	=		—	270
N. 1° W.	×	80	=		—	80
N. 1° E.	×	120	=		120	—
		350			240	350
						240
					350)	110(N. 0° 19′ W.

By the traverse we get N. 0° 19′ W.

There is too much uncertainty in the use of the above method, except in particular cases, to make it generally useful. The traverse table should always be used if at hand.

It is sometimes required that stakes should be set off from the respective angular points, to the

line joining the extreme points, either to save trouble of running a random line or for proving the truth of the operations.

In such case take the difference between each given course and the course of the closing line, noting whether the given line is to the left or right of the closing line, with this difference as a course, and the given distance of the line, take the departures from the traverse table, then the first departure will be the distance of the closing line from the angular point at end of the first distance. The sum or difference of this and the next departure, according as they are both to the same hand or to different hands, will be the distance of the closing line from the end of the second distance, and so proceed to the last point where the closing line and last distance will come together, if rightly done.

Example. Given the bearings and distances of several lines as follows, viz: N. 40° E. 50 perches, N. 38° E. 24 perches, N. 45° E. 40 perches, N. 39° E. 100 perches, required the distance of the closing line from each angular point.

The bearing and distance of the closing line will be found to be N. 40° 15′ E. 213.88 perches. Hence (marking right hand + and left hand —)

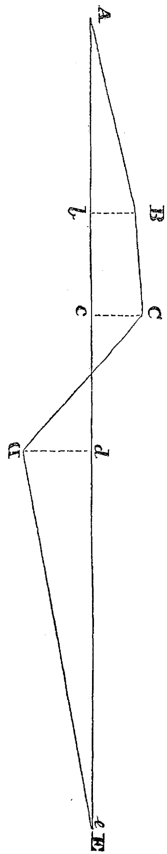
N. E.	N. E.		p.	Lat.	Dep.	p.	
40° 0′	40° 15′	— 15′	50	50.	— .22	— .22	B *b*
38°	"	— 2°.15′	24	23.98	— .94	— 1.16	C *c*
45°	"	+ 4°.45′	40	39.86	3.31	2.15	D *d*
39°	"	— 1°.15′	100	99.98	— 2.18	— 0.03	E *e*
				213.82			

E *e* should, when correctly done, be = 0

N. B. The closing line is not exactly N. 40° 15′ E.; but offsets computed from this line (N. $40\frac{1}{4}$° E.) determine B, C, D, &c., as correctly as if the closing line had been used, and vice versa.

Given A B, N. 10° E. 15 perches, B C, N. 15° E. 19 perches, C D, N. 17° E. 40 perches and D E, N. 12° E. 35 perches, to find the several offsets from the closing line to the angular points A, B, &c.

Ans. The bearing of the closing line running from A, is N. 14° 5′ E. 108.85 perches, the offsets are

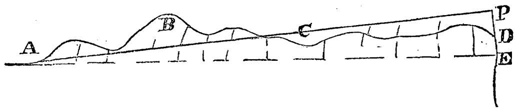


B b = 1.07 perches.
C c = .77 "
D d = 1.26 "
E e = .00 "

Note.—The reverse of the above operation; that is, running a straight line from one extreme point to another along an irregular boundary, and by calculation, finding how far each angular point is from the line so run, may be employed to determine the angular points in the irregular boundary, especially where obstructions render the running on the true line difficult. If the irregular boundary deviates but little from a right line, this method is the most accurate that can be employed.

To straighten a crooked boundary between two estates, so that each estate may have the same quantity of land.

Let A B C D be a crooked boundary. It is required to run a straight line from A to a point P,



in the line passing through D, that shall equalize the quantity of land as before. Run any line A E, near the boundary, and measure perpendicular offsets from this line, to the several bends in the crooked line, and find the areas of the several trapezoids, the sums of which areas will be the area of the irregular figure A B C D E A, which being divided by the half of A E, will give E P, the point P being that to which a line drawn from A will equalize the areas as required.

If the point A should be at a short distance from the boundary, its distance must be taken from the above quotient or added thereto, as the case may require, to obtain E P as before.

If equidistant ordinates or offsets be taken; add together half the sum of the extreme offsets, and the sum of all the intermediate breadths or offsets, which, being multiplied by the equal distance between the ordinates, the product will be the area of the irregular figure as before. Whence the distance E F is found as in the former case.

Example. Let the ordinates or offsets, at six equidistant places be 4, 6, 2, 3, 5 and 8, the equidistance apart being 50.

Here $\frac{4+8}{2} = \frac{12}{2} = 6$ the $\frac{1}{2}$ sum of extremes.

Then $(6 + 2 + 3 + 5) = 22$ and $22 \times 50 = 1100$ area.

Whence $1100 \div (5 \times 50 \div 2) = 1100 \div 125 = 8.8$, and $8.8 - 4 = 4.8 =$ E P, therefore, a line joining the beginning of the boundary near A, with the point P, will fulfill the conditions proposed.

VARIATION OF THE COMPASS.

The irregularity of the variation or declination of the magnetic needle in causing uncertainty in retracing old lines of survey, is well known. A

step towards obviating the errors attending the old method of finding the difference of variation or declination, and thereby obtaining the present magnetic bearing of the lines of survey, has been taken by the Legislature of Pennsylvania, by enacting, that meridian lines should be established in the different counties of the commonwealth, and that surveys hereafter made should be returned according to the true, and not the magnetic bearings of the lines. Every person will at once perceive that much uncertainty in retracing lines will hereafter be removed. Had surveys heretofore been made according to the true bearings, the surveyor would, at the present day, have merely to set the vernier or nonius of his compass to the present variation; then the needle would point out on the face of the instrument the true courses of the lines of survey, by setting it to the proper degree.

All that surveyors would then have to do, would be to go upon the premises to a known corner, and run out the true bearings as given in the title, the bearings shown on the face of the instrument corresponding thereto.

We may deduce the true bearing of a former survey by the following table or accompanying curve, if we know the year the survey was made or lines run. This table was formed by a comparison of the bearings of lines taken at different periods of time. Much difficulty is found in ascertaining the date of survey, formerly made.

Of course this table is given only as an approximation merely. It will serve for places north or south of Philadelphia, and a few miles east or west of the meridian of that place. The table and diagram are sufficiently plain without explanation. The application is as follows. Suppose the magnetic bearing of a line run in 1720 to be N. 45° E., what is the true bearing? We find by the table or diagram the variation in 1720 to be $6\frac{7}{8}$° W., the true bearing of the line is therefore N. $38\frac{1}{8}$° E.

A nonius compass being set as above mentioned to the present variation, and a course run N. $38\frac{1}{8}$° E., will run out the original line.

Again: suppose a line in 1810 bore N. 50° W., what is the true bearing of the line? The variation in 1810 is found to be 2° W., the true bearing

is N. 52° W. The needle, as in the former case, being set to N. 52° W., will run out the original line.

Any person who is at all acquainted with farm surveying, will at once perceive the advantage of surveys being made according to the true courses, and not the magnetic bearings. Ignorant and inexperienced persons will of course object, as more skill and knowledge will be brought into requisition, and therefore their incompetence will be manifested.

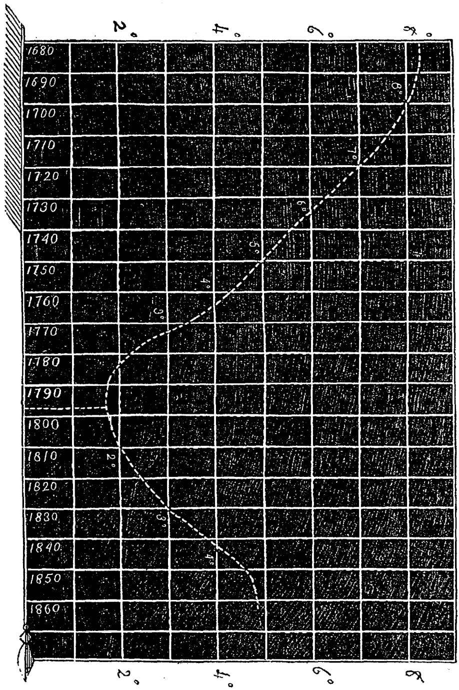
Years.	Variation.
1682,	$8\frac{1}{8}°$ west.
1690,	$8\frac{3}{8}°$ "
1700,	$8°$ to $8\frac{1}{8}°$ west.
1710,	$7\frac{1}{2}°$ to $7\frac{5}{8}°$.
1720,	$6\frac{7}{8}°$.
1730,	$6\frac{1}{4}°$ to $6\frac{1}{8}°$.
1740,	$5\frac{5}{8}°$ to $5\frac{1}{4}°$.
1750,	$4\frac{7}{8}°$ to $4\frac{3}{4}°$.
1760,	$4\frac{1}{8}°$ to $3\frac{3}{4}$.
1770,	$2\frac{7}{8}°$ to $2\frac{5}{8}°$.
1780,	$2\frac{1}{4}°$.

Years.	Variation.
1790,	$1\frac{7}{8}°$.
1800,	$1\frac{7}{8}°$.
1810,	$2°$.
1820,	$2\frac{3}{8}°$.
1830,	$3°$.
1840,	$3\frac{3}{4}°$ to $3\frac{5}{8}°$.
1850,	$4\frac{3}{8}°$.
1852,	$4\frac{3}{8}°$.
1853,	$4\frac{1}{2}°$ west.

N. B. If the surveyor find the variation at his place for any year, the difference between that variation, and the variation found on the chart or diagram, will be a correction which may be applied to variations on the chart, to find the variation, nearly, at his place for any given time.

Years. VARIATION WEST.

TRUE MERIDIAN.



Minimum 1797.

Magnetic curve for lat. 40° 7′ N.—Long. from Philadelphia, 3′ E.

WEST VARIATION.

TABLE OF NATURAL TANGENTS, Radius 1.

Minutes.	0′	10′	20′	30′	40′	50′	Degrees.
1	.0003	.0032	.0061	.0090	.0119	.0148	1.0175
2	.0006	.0035	.0064	.0093	.0122	.0151	2.0349
3	.0009	.0038	.0067	.0096	.0125	.0154	3.0524
4	.0012	.0041	.0070	.0099	.0128	.0157	4.0699
5	.0014	.0044	.0073	.0102	.0131	.0160	5.0875
6	.0017	.0046	.0076	.0105	.0134	.0163	6.1051
7	.0020	.0049	.0078	.0108	.0137	.0166	7.1228
8	.0023	.0052	.0081	.0110	.0140	.0169	8.1405
9	.0026	.0055	.0084	.0113	.0142	.0172	9.1584
10	.0029	.0058	.0087	.0116	.0145	.0175	10.1763

It will be sufficiently exact for calculations in the field, to take the tang. of the degrees and add thereto the tang. of the minutes.

In many calculations the tangents may be used instead of the sines, without material error.

VERSED SINES, Radius 1°.

	0°	10°	20°
1°	$.00015 = \frac{1}{7000}$	$.01837 = \frac{1}{50}$	.06642
2	$.00061 = \frac{1}{3000}$	$.02185 = \frac{2}{90}$	.07282
3	$.00137 = \frac{1}{700}$	$.02563 = \frac{1}{40}$	.07950
4	$.00244 = \frac{1}{500}$	$.02970 = \frac{1}{35}$	.08642
5	$.00381 = \frac{3}{800}$	$.03407 = \frac{1}{30}$	.09368
6	$.00548 = \frac{5}{900}$	$.03874 = \frac{3}{80}$	.10121
7	$.00745 = \frac{3}{400}$	$.04369 = \frac{4}{90}$	.10899
8	$.00973 = \frac{1}{100}$	$.04894 = \frac{1}{20}$	.11705
9	$.01231 = \frac{1}{80}$	$.05448 = \frac{5}{90}$	.12538
10	$.01519 = \frac{1}{70}$	$.06031 = \frac{3}{50}$	.13397

Multiply the versed sine of the elevation of the hill by the distance of the slope or surface measure, which, being deducted from the slope or oblique measure, gives the horizontal distance.

Example. The oblique distance of a hill of 5° elevation is 40 perches. What is the horizontal measure?

Here $.00381 \times 40 = .1524$, this deducted from 40, gives 39.85 $(40 - .15)$ per. the distance required.

Or, because $.0038 = \frac{3 \cdot 8}{100}$ nearly, we have $40 - \frac{3 \times 40}{800} = 40 - .15 = 39.85$ per. as before.

Note. The numbers in this table might have been expressed by vulgar fractions as in the foregoing example, which would, in the field, be preferable to the decimal form, in many instances abridging the calculations, and yet be sufficiently correct. This the surveyor can readily perform for his own use.

The oblique measure of any line may be reduced to horizontal measure by the traverse table, in the following manner: under the degrees of elevation or depression, and opposite the oblique distance in the distance column, a number will be found in the lat. column, which is the horizontal distance; the number in the departure column being the vertical altitude of the hill or slope. Taking the preceding question, we have, under 5° and distance 40, the number 39.85 in a lat. column, which is the horizontal distance as before.